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The Finite-Element Model Creation of a Gear Segments of the Locomotive Traction Gear

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1. Introduction

The present work is continuation of the article [1] in which the solving procedure of the contact interaction of the teeth of a gearings based on the application of the FEM has been offered. The specified technique was successfully applied in a case of plenty of elements on the evolvent profile of the teeth for flat model. However for the spatial problems of the teeth contact it is necessary to limit to relatively few quantity of finite elements along evolvent profile taking into account a calculation time. It is necessary to guarantee a formation of the node contact pairs for decreasing of an error of the calculations.

2. Creation of FE-model of a gear segment

At calculation of any mechanical units on a method of final elements the first arising problem is the problem of geometrical modeling. The given problem has already been considered in the previous work [2]. However the problems connected with geometrical modeling of gearing do not terminate on transfer to program FE - the analysis of evolvent profile of a tooth set pointwise. It is very difficult to work with such profile as the profile of a gear segment of three teeth which consist of several hundreds points which should be processed manually. Therefore the program has been modified for transfer of geometry of teeth as a contour consisting of several curves. This step is dependent on the further processing of a neutral file with the results of the account of the program. As for the further work pre/postprocessor FEMAP has been elected the form of final curves which are necessary for using for approximation received before points of a contour of evolvent tooth should be chosen from limited assortment of applicable curves. These are direct lines (obviously, that the structure constructed from pieces of straight lines only is vaguely similar to a real structure and consequently it is inapplicable for the given work), arches of circles (more correct form of the curves replacing a real contour, however a problem of definition of radius of curves arises as it should be variable, the problem of definition of coordinates of the centers of arches follows from here). The elliptic, parabolic and hyperbolic forms of approximating curves are also possible, however any of them has a number of lacks which do not allow to use them. The choice has been made for the benefit of the special curves named non-uniform rational Bezier splines - Non Uniform Rational Bezier Splines (NURBS) .But when there are too many approximated points, mathematical task of Bezier splines is a very tiresome and toilful process if it is possible. It was possible to solve this problem by more detailed consideration of structure of Bezier splines. These splines are of several types. As the analysis of the engineering specifications has shown, usual Bezier splines are curves of the third order. However there are the simplified splines of the second order and they are a special case of complex multipoint NURBS. For their construction it is enough to have only three control points, and at the correct task of control points for the simplified splines the difference between the received curves is so insignificant, that it can be neglected without any damage for considered model. It is known that circles, ellipses, parabolas and hyperboles are conic sections which can be expressed mathematically as functions of the second order. Rather useful property of simplified Bezier splines consists in depending on the coordinates of control points these splines can become any conic section as the last are a special case of Bezier splines. Three-dot Bezier splines are curves of the second order for which mathematical dependence of tensor $\{P(t)\}=\begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix}$ can be submitted as

$$\{P(t)\}=(1-t)^2\{P_0\}+2t(1-t)\{P_1\}+t^2\{P_2\}, \quad (1)$$

Where the parameter t changes in limits $0 \le t \le 1$. But the principal cause on which simplified Bezier splines were chosen is an opportunity of the further mathematical operating above them have been chosen for reception, for example, exact value of length of curves, etc. The detailed procedure of the description of a structure by Bezier splines has been described in work [3].

The following and rather important stage in creation finite element of gear segment is the splitting of a contour into the certain sites in control points of which units finite element mesh will be located further. This stage is necessary, as the accuracy of received results is pawned on the given step. Adequacy of results to real fields of pressure and deformations depends on a regularity received in result FE mesh in teeth of a considered segment. Besides contact problems were solved earlier, however always there is a problem connected with an inequality of central forces in zones of contact. The problem is, that if contact between FE models of bodies is carried out through the coordinated pairs contacting units central forces are equal among themselves, and the received distributions of pressure adequately reflect physics of process [4]. If central contact it is not provided, the inadequate decision will be result as it is shown in fig. 1.

It was possible to expect concurrence of units in a zone of contact without corresponding calculations only on occasions. That such "accident" law became necessary generation finite element mesh of cooperating cogwheels which should be carried out so that at modeling rotation of cogwheels contact pairs units would be formed. Exact calculation of coordinates of central points is necessary for such generation. It is necessary to take into account, that for spatial problems of force contact teeth it is necessary to be limited relatively to a small amount of final elements lengthways evolvent profile.

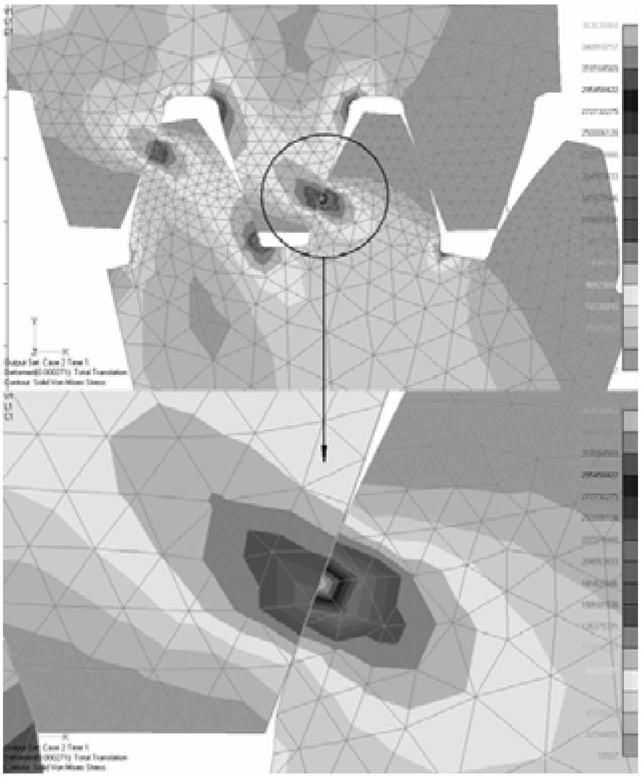

Fig. 1. Distribution of equivalent pressure by criterion Von Mises at discrepancy of contact units (the bottom half of figure is the increased image of a stain of contact)

To solve this problem we shall consider a pinion gear segment, consisting of three teeth. Pinion rotation from initial to final position will be considered, they will be defined by the moments of an entrance and an exit from gearing the central (basic) tooth of pinion. We shall name other two teeth the first and the second additional teeth. The first additional tooth is in gearing in an initial phase of modeling, according to the second one an additional tooth is in a final phase. Teeth of wheels are similarly considered.

We shall consider the basic contact pair of teeth and we shall define its initial and final positions of the basic contact pair of teeth. For this purpose it is necessary to model pinion rotation from initial to final position so that it turned on a constant corner of turn. The total quantity of pinion turns should correspond to planned number of elements on a working part evolvent profile. Thus for each intermediate position considered teeth points of crossing of the current structure and a line of gearing will define the position of contact units for projected FE mesh. It is obvious, that in initial and final positions of teeth, two extreme units are defined on a working surface of evolvent. On fig. 2 the modeling of rotation of the basic tooth pinion is shown.

Its binding to the central corner of turn of the structure is necessary for unequivocal definition of the current position of a structure of a tooth, determined on an initial circle of pinion. Initial circles are chosen because at any parameters of gearing initial circles "slide" the each by other. The working part of a line of gearing is defined by points L_1 and L_2, which correspond initial (the moment of an entrance in contact) and final (the moment of an exit from gearing) to positions of a tooth. Thus positions are defined by initial $\alpha_s = \angle W_1 O_1 P$ and final $\alpha_f = \angle W_2 O_1 P$ corners. These corners are the central corners of an initial circle of pinion, as W_1 and W_2 - points of crossing of a structure of a tooth with an initial circle of pinion for initial and final position accordingly, and P - a pole of gearing.

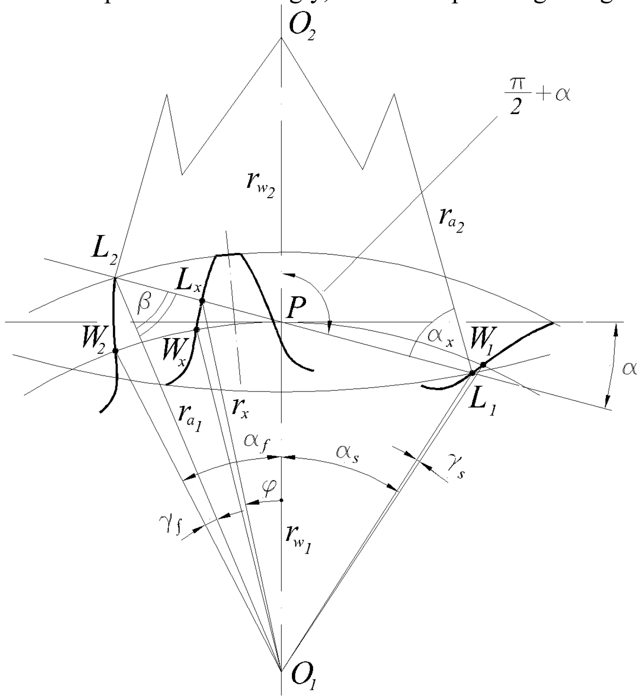

Fig. 2. Modeling of rotation of the basic (central) tooth of a segment of pinion

Corners α_s also α_f can be found as the sum of corners

$$\alpha_s = \angle PO_1L_1 + \gamma_s . \ \alpha_f = \angle PO_2L_2 + \gamma_f \qquad (2)$$

For the determination of the specified making corners it is necessary to define radiuses of initial circles of wheels r_{w1} and r_{w2} as in case of nonzero total displacement χ_Σ radiuses pitch and initial circle are not equal to each other [5]

$$r_{w1} = \frac{mz_1}{2} \cdot \frac{\cos(\alpha_d)}{\cos(\alpha)} \ r_{w2} = \frac{mz_2}{2} \cdot \frac{\cos(\alpha_d)}{\cos(\alpha)}, \qquad (3)$$

Where m - the module of teeth, z_1 - quantity of pinion teeth, z_2 - quantity of wheels teeth, α_d - a corner of a structure of an initial contour (section of rack by a plane, perpendicular to a direction of teeth), α - the angle of action (at corrected wheels at $\chi_\Sigma \neq 0$ $\alpha \neq \alpha_d$).

The corner α_x is calculated with use of the theorem of sine for a triangle PL_1O_2 (fig. 2)

$$\sin(\alpha_x) = \frac{r_{w2}}{r_{a2}} \sin\left(\frac{\pi}{2} + \alpha\right), \qquad (4)$$

where r_{a2} - radius of addendum circle of wheel.

The length of a part of a line of gearing from a pole P up to a point L_1 is with use of the cosine rule

$$PL_1 = \sqrt{r_{w2}^{\ 2} + r_{a2}^{\ 2} - 2r_{w2}r_{a2}\cos\left(\frac{\pi}{2} - \alpha - \alpha_x\right)}. \quad (5)$$

It is possible to find a piece O_1L_1 from a triangle PL_1O_1, again using the cosine rule and already found value PL_1,

$$O_1L_1 = \sqrt{r_{w1}^{\ 2} + PL_1^{\ 2} - 2r_{w1}PL_1\cos\left(\frac{\pi}{2} - \alpha\right)}, \quad (6)$$

Then it is possible to calculate a required corner $\angle PO_1L_1$ from the formula

$$\sin(\angle PO_1L_1) = \frac{PL_1}{O_1L_1}\sin\left(\frac{\pi}{2} - \alpha\right). \quad (7)$$

Similarly we determine a corner

$$\angle PO_1L_2 = \frac{\pi}{2} - \alpha - \arcsin\left[\frac{r_{w1}}{r_{a1}}\sin\left(\frac{\pi}{2} + \alpha\right)\right]. \quad (8)$$

Additional corners γ_s and γ_f are defined by a structure of a tooth. Each of these corners is formed between two radiuses - vectors which have been carried out from a point O_1 to two points on evolvent profile, thus one of radiuses - vectors is carried out to the point W_i. Fig. 3 represents the increased image of a pinion tooth in the current position and is determined by a corner φ can explain a technique of a determination of additional corners. The required corner γ_x can be found as half-difference of corners η_w and η_x

$$\gamma_x = \frac{1}{2}(\eta_w - \eta_x), \quad (9)$$

Each of which is as the ratio of corresponding thickness of a tooth to radius of a point on evolvent profile

$$\eta_w = \frac{s_{w1}}{r_{w1}}, \qquad \eta_x = \frac{s_x}{r_x}. \quad (10)$$

For the determination of thickness of a tooth s_x it is possible to take advantage of the known formula [6]

$$s_x = \left\{\frac{s_{d1}}{r_{d1}} - 2\left[inv\left(\arccos\frac{r_{b1}}{r_x}\right) - inv(\alpha_d)\right]\right\} \cdot r_x, \quad (11)$$

Where r_x - the current radius on evolvent profile, r_{d1} - radius pitch circles of pinion, s_{d1} - width of a tooth on pitch circles, r_{b1} - radius of the base circle of pinion.

Thickness of a tooth s_{w1} is determinated similarly. It is obvious, that for definition of considered additional corners γ_s and γ_f as the current radiuses r_x it is necessary to use the piece O_1L_1 which was found earlier and radius addendum circle of pinion r_{a1}, accordingly. Thus, it is possible to count, that initial α_s and final α_f corners are determined.

Points L_x for each current position of a structure will define an arrangement of units of projected FE mesh. Thus positions of a structure are defined by a corner of turn φ in conformity about fig. 2. The corner φ changes from $-\alpha_s$ up to α_f with increment Δ which is under the formula

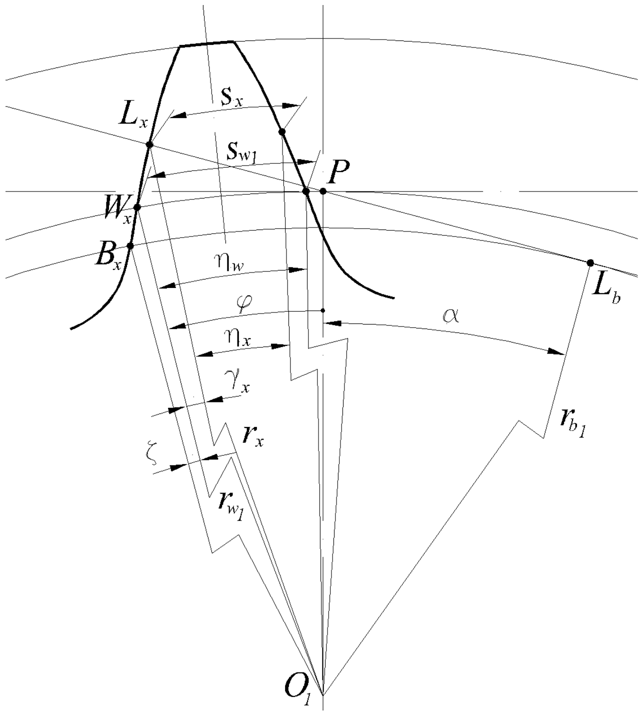

Fig. 3. Definition of additional corners

$$\Delta = \frac{\alpha_s + \alpha_f}{n}, \quad (12)$$

Where n is quantity of elements on a working part evolvent profile determined in initial dialogue of the program.

For the determination of coordinates of units FE mesh we shall determine radiuses r_x of points L_x for the current positions of teeth. From a triangle $L_xL_bO_1$ it is possible to determine

$$r_x = \sqrt{L_xL_b^{\ 2} + r_{b1}^{\ 2}}, \quad (13)$$

where r_{b1} is radius of the base circle of pinion.

For the determination of a piece L_xL_b we shall take advantage of property of evolvent $L_xL_b = \cup B_xL_b$. For the determination of an arch of the basic circle $\cup B_xL_b$ it is necessary to determine a corner between its radiuses O_1B_x and O_1L_b

$$\angle B_xO_1L_b = \alpha + \varphi + \zeta, \quad (14)$$

Where the additional corner ζ is by the technique described above with use of the formula (9). The determination of this corner is similar to the determination of a corner γ_x for what it is necessary to replace only a point L_x on B_x.

Boundary units of the FE mesh are under construction for the central arrangement of a tooth of pinion. Assuming, that the tooth is in the central position and is symmetric concerning an axis of ordinates, we count, that the unit will be in a point of a structure which r_x radius is earlier found. Then coordinates of a point on the profile are from a rectangular triangle where r_x radius of which was found earlier. Then coordinates of a point on the profile are from a rectangular triangle where r_x is a hypotenuse. The sharp corner ψ with top in the center on an axis of rotation of a cogwheel is determined as

$$\psi = \frac{s_x}{2} r_x , \qquad (15)$$

whence coordinates of a contact point on a structure can be calculated as

$$x = r_x \sin(\psi) \;\; y = r_x \cos(\psi) . \qquad (16)$$

According to the modeling of rotation described above teeth of pinion calculations repeat so much time, how many units are set on a working part evolvent profile then record in a neutral file of a structure of the tooth consisting of contact points of the first tooth is organized.

As it has been told earlier, the gear segment of pinion, consisting of three teeth is considered. As the factor of gearing e for spur gearings is in limits from 1.3 up to 1.7 [5] if the basic tooth of a gear segment should turn from initial to final position then extreme teeth will also get in area of gearing partly. If construction of the geometry additional teeth is possible, as it, as a matter of fact, only turn control points describing profile of the basic tooth on an angular step $2\pi/z_1$ (fig. 4) the determination of contact central points on these teeth demands additional calculation as central points on the basic and additional teeth, unfortunately, do not lay on circles of the same radiuses.

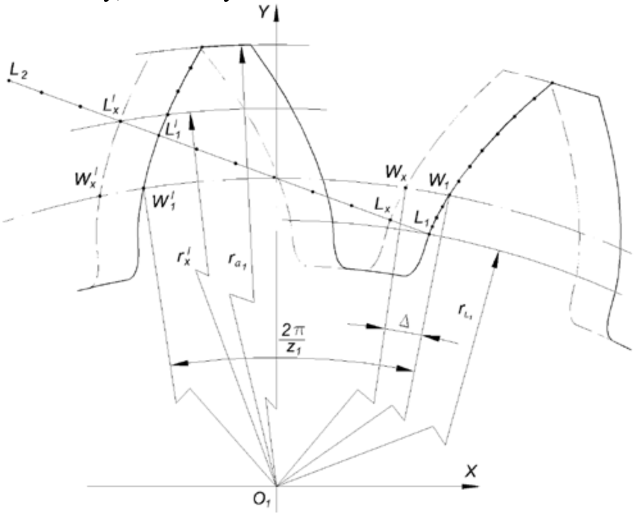

Fig. 4. Modeling of construction of contact central points on the first additional tooth

Let's carry out modeling of generation of contact central points on the first additional tooth. On fig. 4 continuous thick line shows position of a gear segment at the moment of an entrance in contact of the basic tooth of pinion. According to the technique described above for the given position two characteristic points can be determined: a point W_1 laying on pitch circle and a point L_1 which lays on crossing of a structure of a tooth and a line of gearing. The point L_1 is an initial contact point for the basic tooth. The similar point $L_1^/$ is an initial contact point for the first additional tooth. For its determination it is necessary to determine a point $W_1^/$. It is obvious, that last will be distant from a point W_1 on pitch circle on circular pitch of gearing. Its coordinates are determined by the following formulas:

$$W_{1x}^/ = -r_{w1} \cdot \sin\left(\arctan \frac{W_{1x}}{W_{1y}} + \frac{2\pi}{z_1} \right), \qquad (17)$$

$$W_{1y}^/ = r_{w1} \cdot \cos\left(\arctan \frac{W_{1x}}{W_{1y}} + \frac{2\pi}{z_1} \right), \qquad (18)$$

Where W_{1x}, W_{1y} are coordinates of a point W_1. It is necessary to note, that in brackets there is a difference of corners, as in the chosen system of coordinates $W_{1x} < 0$.

According to the technique described above if the current position of a structure of a tooth is known that is defined by coordinates of a point $W_1^/$, crossing of a structure of a tooth with a line of gearing that allows to find coordinates of a required first contact of central point $L_1^/$ can be found. Carrying out rotation of a gear segment on the set corner of turn Δ, points $W_x, W_x^/$ and $L_x^/$ (fig. 4) can be consistently found. We shall note, that the specified algorithm should be limited to a condition $r_x^/ \le r_{a1}$. Thus, contact central points for the first additional tooth are constructed. These points are only on a part evolvent profile, on other part any splitting can be used.

Construction of contact central points for the second additional tooth is carried out similarly, with that difference, that the current radius on evolvent profile for the second tooth should be more, than radius of a circle of the beginning of a working part of a structure, i.e. $r_x^{//} \ge r_s$.

At last, coordinates of all found contact points can be written down in a neutral file. Thus, the structure of a gear segment is set by Bezier splines, and splitting of active parts of evolvent can be carried out manually with the help of the exported contact points.

3. Calculation of the received FE-model

The received models for each of gear segments (a wheel and pinion) it is consistently imported in FEMAP as neutral NEU files. These files are an open format for record and converting of the objective data, developed by company Structural Dynamic Research Corp. And completely we compose to the world standards. In FEMAP final cosmetic completion of geometrical model, the task of properties of materials, elements and generation of a finite - element grid is made if necessary. It has been established by practical consideration, that at the given stage of development of the software by the best program for the decision of nonlinear problems with an essential degree of nonlinearity, namely to such contact problems concern, program MSC.MARC is. Therefore the received model is imported in *.dat a file - a standard file for data transmission to program MARC. After importation of model it is necessary to enter only necessary boundary conditions and to send a problem on the account. The result of FE-calculation can be seen on fig. 5 in program MARC. Here the intense condition (equivalent pressure by criterion Von Mises) for traction gearings of electric locomotive EU07 is considered.

4. Conclusions

As a result of the carried out calculations it is possible to draw a conclusion that the specified finite - element design procedure tensely - deformable conditions spur gearings is developed. It can be effectively used for the analysis of work large-module gearings of traction engines of locomotives, miner mills and other equipment.

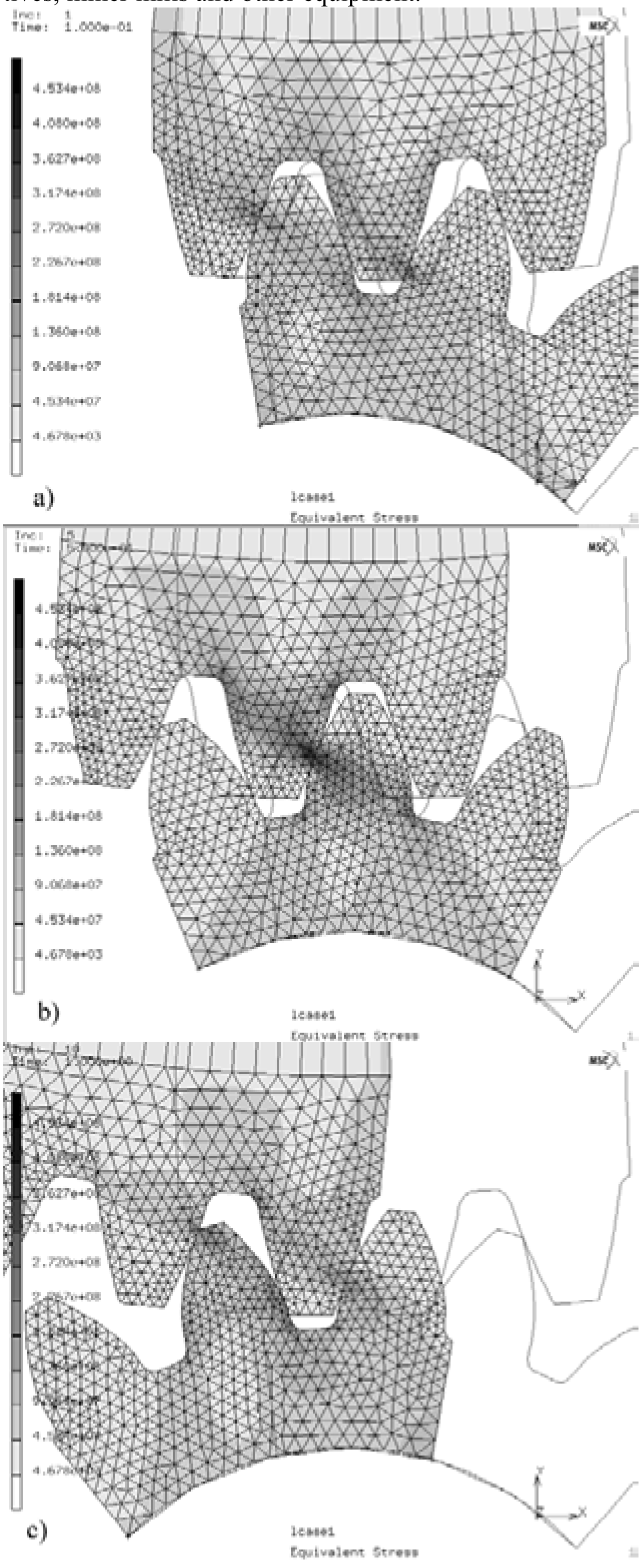

Fig. 5. An example of calculation
of stress-deformed state of a gearing segment

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THE FINITE-ELEMENT MODEL CREATION OF A GEAR SEGMENTS OF THE LOCOMOTIVE TRACTION GEAR

S u m m a r y

In the presented work the technique of creation of finite - element model of the gear segment consisting of three teeth is submitted. As the practical application of the given technique calculation of traction transfer of universal electric locomotive EU07 is carried out.

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СОЗДАНИЕ КОНЕЧНО-ЭЛЕМЕНТНОЙ МОДЕЛИ ЗУБЧАТОГО СЕГМЕНТА ТЯГОВОЙ ПЕРЕДАЧИ ЛОКОМОТИВА

Р е з ю м е

В настоящей работе представлена методика создания конечно-элементной модели зубчатого сегмента, состоящего из трех зубьев. В качестве практического применения данной методики проведен расчет тяговой передачи универсального электровоза EU07.

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