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CONTENTS

ANALYSIS OF CALCULATION ERRORS IN FEM

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Finite Element Method (FEM), Calculation Errors, Stresses in a Disk

Introduction

Now on railways of the world a plenty of railway wheels of various constructions is used. They carry out a role of an initial elastic element in system a track - a rail - a wheel - a suspension bracket - a body. It causes increased requirements to mechanical properties of wheels by way of their reliability. Among these wheels the most part is carried out as seamless-rolled or cast, and the wheel as a unit is conditionally broken on centre of a wheel, disk and rim. Thus the most deformable part is the disk of a wheel. The basic problems of destruction of wheels as defects of other parts seldom result in catastrophic consequences are connected with its serviceability.

The most part of constructive updating of wheels is connected to change of the form of their disks. It causes such steadfast attention to operation of these parts of wheels at various kinds of their loading. It is known, that railway wheels are loaded with various loading which has a axially symmetric as well as non axially symmetric character. Therefore now wheels are usually calculated by means of a method of final elements as three-dimensional bodies.

The analysis of such calculations, for example, by results of the last 13 Congresses on wheel pairs (Rome, 2001), has shown, that for such calculations basically the standard software packages realising FEM are used. Among such packages the greatest popularity use ANSYS, COSMOS, NASTRAN. All these packages have an opportunity of automatic generation certainly - an element grid. However if the grid for all wheel as a whole is created FE, such generators do not give an opportunity for the differential approach. I.e. splitting rim parts on elements of one size, centre of a wheel - another etc. In result such approach is frequently carried out, that all parts of a wheel are broken into elements about one size. However to break all wheel on elements small enough is not possible, as in this case the amount of units and, accordingly, degrees of freedom of model sharply grows.

Result of the described approach is rather small (from 1 up to 4) amount of elements on thickness of a disk. Earlier in the operations [1] authors specified, that at such approach reception of an inadequate solution of a problem is probable. The error of it may run to 150 % and higher.

The present operation is devoted to the analysis of an error in calculations which may arise at use various FE grids and, accordingly, definition optimum on step-type behavior and time of calculations for FE discretization.

For analysis of an error of design circuits FEM it is necessary to have any analytical solution as test. Thus it is necessary, that such solution would be as much approximate as possible to a real problem of deformation of railway wheels. It is known, that disks of wheels are subjected to the complex intense condition which may be conditionally divided on mainly flat and mainly bent. Accordingly, as test design model deformation of a flat disk of constant thickness is considered at action by flat or bent loading.

Test problem 1

As the first test problem we shall consider deformation of a flat disk of constant thickness h (fig. 1). The considered disk on the circuit is designated 1. Its external contour of 2 radiuses r_2 we count immobile and not deformable, and its internal contour of 3 radiuses r_1 let it be dislodged as the rigid whole on preset size Δ_y.

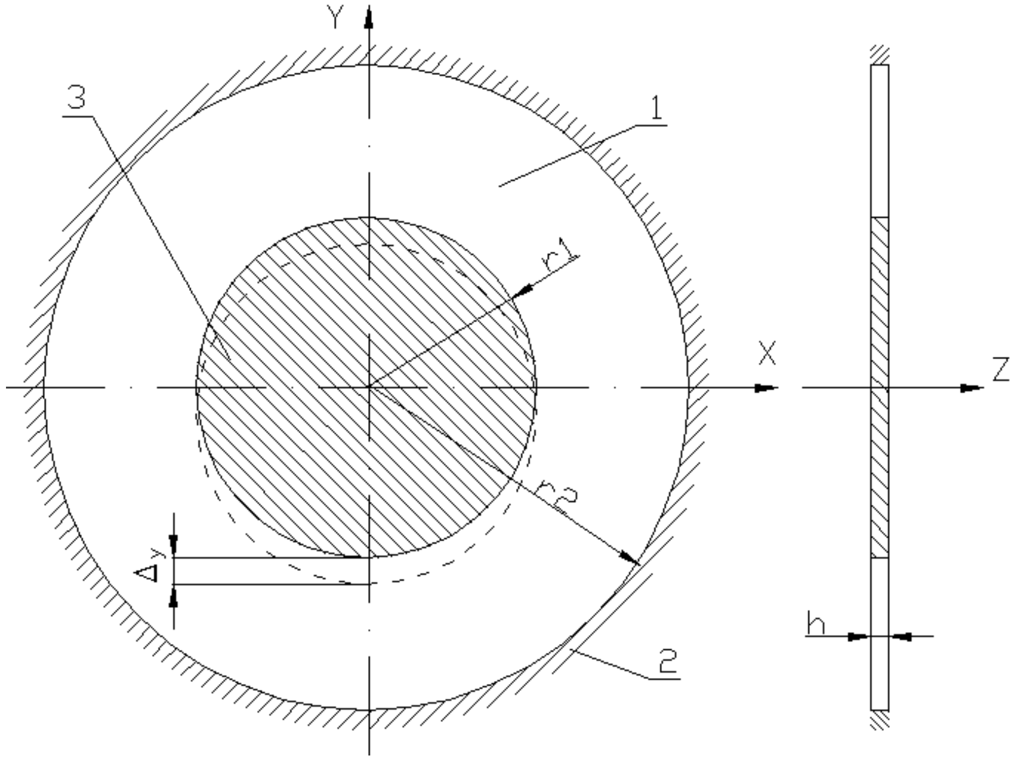

Fig. 1. The design circuit of a test problem 1

The solution of the specified problem was found earlier by authors in operation [2]. If to consider a problem, as a certain analogue of a real wheel, for example, GOST 9036-88, calculation of the maximal radial stresses σ_r for it was carried out with use of package Mathcad 2000 Professional. The appropriate report of a solution is shown as fig. 2. The maximal tensile stresses equal 142,8 MPa thus are reached in the points which are taking place in the top part of an internal mobile contour of a disk.

The numerical solution of a problem with help FEM was received with use of package MSC/NASTRAN for Windows release 4.5. On fig. 3 FE discretization of disk area (a grid 3x5x80) and boundary conditions preset at it is given.

As we see from the given figure, the FE grid using prismatic eight-node elements such as SOLID was used. For boundary nodes on an external contour complete fixing was preset, the same was executed for nodes on an internal contour, but according to requirements NASTRAN displacement on vertical coordinate in addition was preset.

On fig. 4 the solution (a field of stresses σ_r) for considered area is given. Results of calculations for various FE grids are given in table 1. Here discretization of considered area is resulted in the first column, and the first parameter - amount of nodes on thickness of a disk, the second - on radius, the third - on district coordinate.

$r_1 := 0.19$ m $\qquad r_2 := 0.36$ m $\qquad h := 0.02$ m

$E := 2 \cdot 10^{11}$ MPa $\qquad \nu := 0.3 \qquad \Delta_y := 0.0001$ m

$$\kappa := \frac{3-\nu}{1+\nu} \qquad m := \frac{E}{2 \cdot (1+\nu)} \qquad c := r_1^{\,2} - r_2^{\,2} - \kappa^2 \cdot \left(r_1^{\,2} + r_2^{\,2}\right) \cdot \ln\left(\frac{r_1}{r_2}\right)$$

$$\sigma_r := m \cdot \frac{\Delta_y \cdot \left[3 \cdot \kappa \cdot \frac{\left(r_1^{\,2} + r_2^{\,2}\right)}{r_1} + 2 \cdot r_1 + \kappa^2 \cdot \left(\frac{r_1^{\,2} + r_2^{\,2}}{r_1}\right) - 2 \cdot \kappa \cdot \frac{r_2^{\,2}}{r_1}\right]}{c}$$

$$\sigma_{max} := \frac{\sigma_r}{1000000} \qquad \sigma_{max} = 142.764 \quad \text{MPa}$$

Fig. 2. The report of a solution of a test problem 1 with help Mathcad 2000 Professional

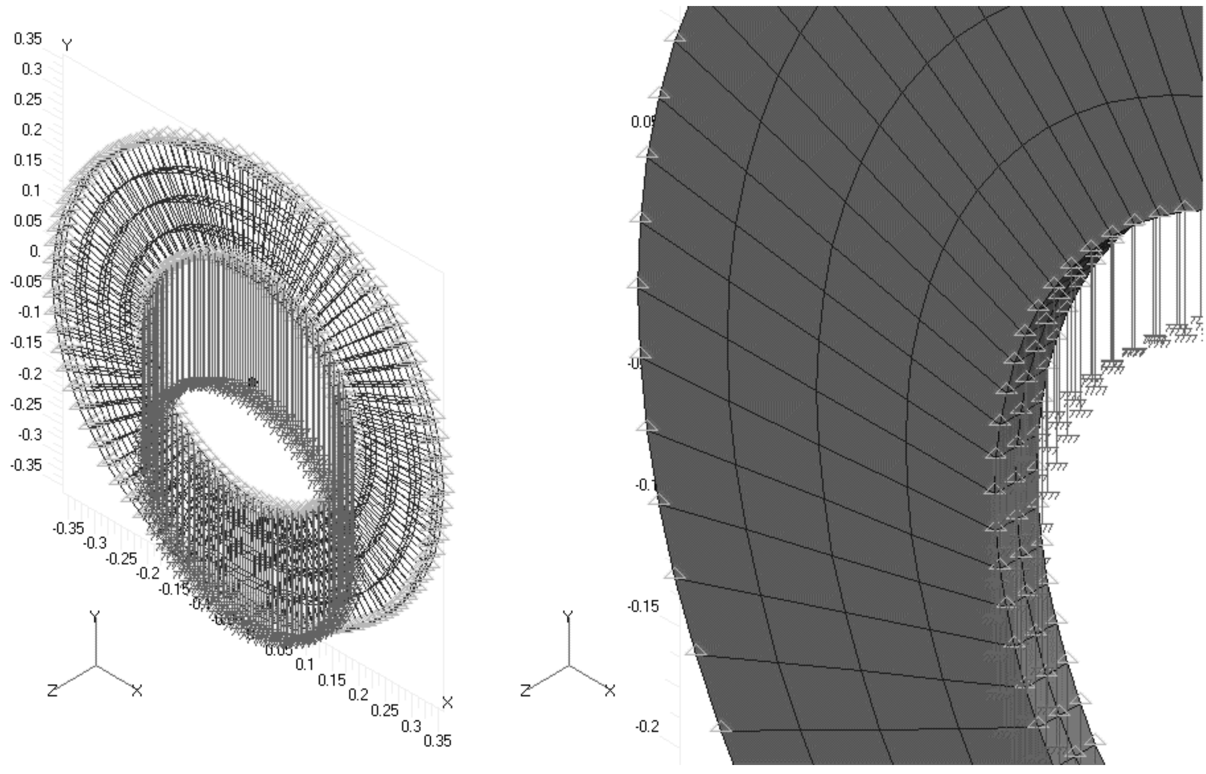

Fig. 3. FE discretization of a disk and the task of boundary conditions

Apparently from reduced results of calculations, at magnification of discretization of a FE grid from 3x5x20 (300 nodes) up to 6x20x80 (9600 nodes) processor time of the score is augmented from 1,4 with up to 105,5 with. Unfortunately, proportionality here it is not observed, and time of the score is augmented more promptly. The strange thing is observed also at first sight. At magnification of a discretization of a grid the error of a numerical solution increases.

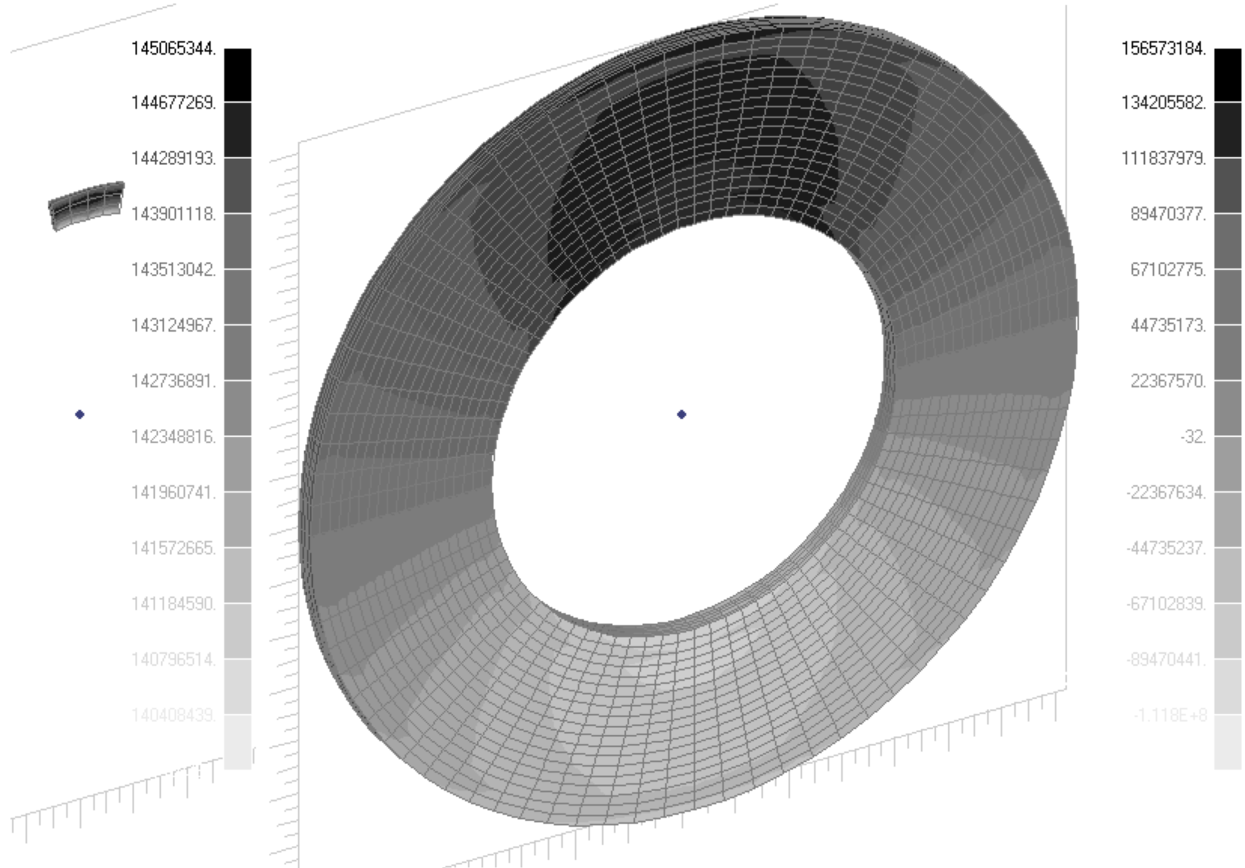

Fig. 4. A field of radial stresses for a test problem 1

The table 1

FE grid	Nodes / Elements	Ratio of lengths of the sides of an element Lt/Lr min	Ratio of lengths of the sides of an element Lt/Lr max	Problem 1 σ_r max (MPa)	Processor time of one step (s)
Analytical solution				143,9	
3x5x20	300/160	1,40	2,65	142,8	1,4
3x5x40	600/320	0,702	1,33	144,1	2,3
3x5x80	1200/640	0,351	0,665	144,4	4,6
3x20x20	1200/760	6,65	12,6	149,3	6,2
3x20x40	2400/1520	3,33	6,31	151,4	11,3
3x20x80	4800/3040	1,67	3,16	152,0	28,1
6x20x20	2400/1900	6,65	12,6	154,0 147,6	20,4
6x20x40	4800/3800	3,33	6,31	156,1 145,8	49,6
6x20x80	9600/7600	1,67	3,16	156,6 145,1	105,5

This phenomenon has found the reflecting when such known phenomenon as an edge effect was analyzed. Really, increase of an error of evaluations is bound to it as if to select a band (elements) in which stresses are peak, and among them to consider those devices, which are on middle (far from exterior lateral areas of a disk), on results are gained much closer to an analytical solution which also does not take into account an edge effect. On fig. 4 the stress distribution in such devices at the left is shown. Accordingly, in table 1 in the fifth column in last three cases two numbers are given. Thus first takes into account all tension, including an edge effect, and second - without taking into account an edge effect.

Thus, is visible that the magnification of a discretization of a grid on environing coordinate though, eventually, reduces an error of evaluations, but not strongly influences it. At the same time the discretization on two other coordinates has essential effect on an error of evaluations. As a whole it is possible to tell, that for prolated devices (that parameters in 3 and 4 column characterize) which from position FEM theories are intolerable, the solution of a test problem 1 may be obtained with a margin error evaluations not exceeding 5 % even for FE grids with a small discretization.

Test problem 2

Let's consider the following test problem which represents the same area which strain is curving. In this case the interior contour is dislodged as the rigid whole in a transverse direction, causing rotationally symmetric curving of a disk (fig. 5).

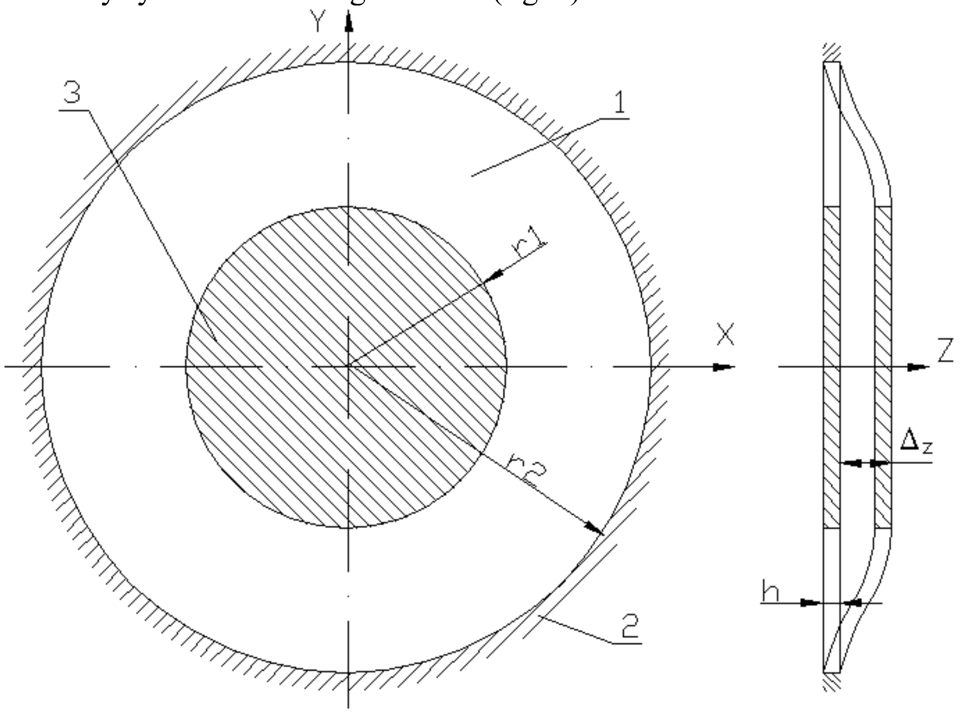

Fig. 5. The design scheme of a test problem 2

The solution of this problem also was found in operation [2], and its embodying with use of package Mathcad 2000 Professional is shown on fig. 6, and one of its numerical embodying (a grid 3x20x20) is submitted on fig. 7.

For the given problem calculations with use FEM for different discretization of FE grids (table 2) were also carried out. However results for grids with a small discretization are not given, as the error of evaluations for them is very great also such grids may not be used by designers in the calculations.

$r_1 := 0.19\ \text{m} \qquad r_2 := 0.36\ \text{m} \qquad h := 0.02\ \ \text{m}$

$E := 2 \cdot 10^{11}\ \text{MPa} \qquad \nu := 0.3 \qquad \Delta_y := 0.0001\ \ \text{m}$

$$\kappa := \frac{3-\nu}{1+\nu} \qquad m := \frac{E}{2\cdot(1+\nu)} \qquad c := {r_1}^2 - {r_2}^2 - \kappa^2 \cdot \left({r_1}^2 + {r_2}^2\right) \cdot \ln\left(\frac{r_1}{r_2}\right)$$

$$\sigma_r := m \cdot \frac{\Delta_y \cdot \left[3\cdot\kappa\cdot\frac{\left({r_1}^2+{r_2}^2\right)}{r_1} + 2\cdot r_1 + \kappa^2\cdot\left(\frac{{r_1}^2+{r_2}^2}{r_1}\right) - 2\cdot\kappa\cdot\frac{{r_2}^2}{r_1}\right]}{c}$$

$$\sigma_{max} := \frac{\sigma_r}{1000000} \qquad \sigma_{max} = 142.764\ \ \ \text{MPa}$$

Fig. 6. The report of a solution of a test problem 2 with help Mathcad 2000 Professional

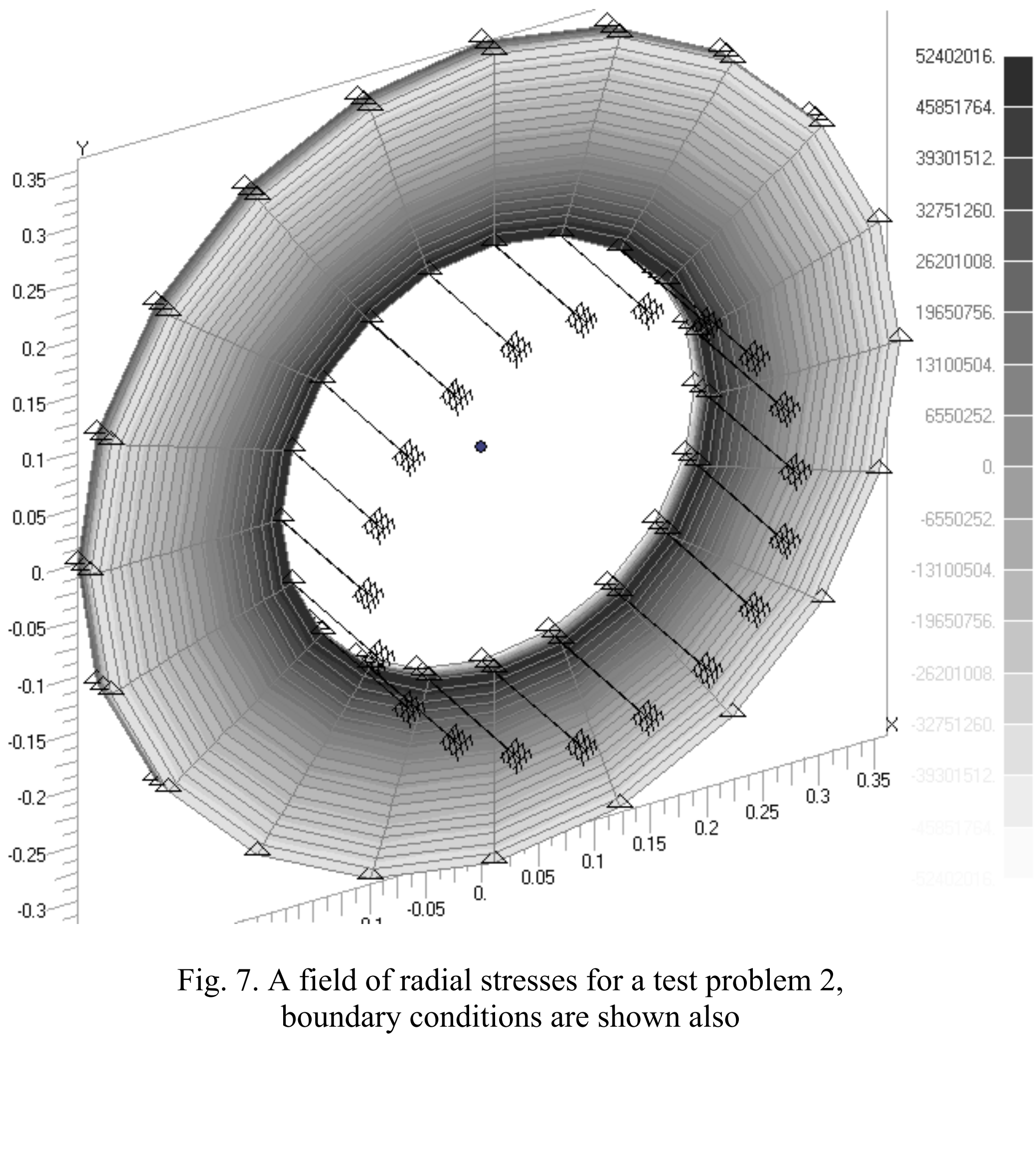

Fig. 7. A field of radial stresses for a test problem 2,
boundary conditions are shown also

The table 2

FE grid	Nodes / Elements	Ratio of lengths of the sides of an element Lt/Lz	Ratio of lengths of the sides of an element Lr/Lz	Problem 2 σ_r max (MPa)	Processor time of one step (s)
Analytical solution				57,72	
3x20x20	1200/760	11,3	0,895	52,40	6,2
3x20x40	2400/1520	5,65	0,895	51,53	11,3
3x20x80	4800/3040	2,83	0,895	51,29	28,1
6x20x20	2400/1900	22,6	1,79	55,23	20,4
6x20x40	4800/3800	11,3	0,895	54,29	49,6
6x20x80	9600/7600	5,65	0,895	54,03	105,5
12x20x20	4800/4180	45,2	3,58	56,38	72,4
12x20x40	9600/8360	22,6	1,79	55,41	244,1
12x20x80	19200/16720	11,3	0,895	55,15	420,7

The analysis of results, reduced in table 2 displays, that the loading of disks curving is more critical to choice of FE discretization. Thus the most essential is the major discretization of a grid on width of a disk that was unessential for the first test problem. In the least degree the exactitude of a solution of a problem depends on a discretization of a grid on environing coordinate.

Resume

To apply FEM for the analysis of stressed - deformed states of railway wheels it is necessary to investigate in advance an error of the evaluations stipulated by FE discretization. It is defined, that the peak error to a solution is imported with a small amount of nodes on width of a disk. However use of the uniform discretization obtained, for example, by means of self-acting generators of a FE grid, in this case is impossible, as it gives in very major number of freedom degrees of a model.

It is the most expedient to select FE grids with number of nodes on width of a disk from 6 up to 12, that ensures an error of evaluations up to 10 % in the first case or up to 5 % - in the second. Also it is desirable to provide good discretization of a FE grid on radius of a disk (20 - 40 nodes). Rather small amount of nodes on environing coordinate does not import an essential error to a solution of a problem.

The literature

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Sitarz M., Sładkowski A., Chruzik K. Analysis of calculation errors in FEM
The analysis of influence finite element digitization on an error of evaluations is carried out at examination of a tension of direct disks of a stationary value of width. The problem is tested on two analytical solutions. It may be used at FE calculation of seamless-rolled sprockets of coaches and locomotives. It is defined, that the peak error to a solution is imported with a small amount of nodes on width of a disk. Use of the uniform digitization in this case is impossible, as it gives in very major number of freedom degrees of a model. Rather small amount of nodes on environing coordinate does not import an essential error to a problem solution.

Ситаж М., Сладковский А., Хружик К. Анализ погрешности вычислений при помощи МКЭ
Проведен анализ влияния конечно элементной дискретизации на погрешность вычислений при исследовании напряженного состояния прямых дисков постоянной толщины. Задача тестирована на двух аналитических решениях. Может быть использована при КЭ расчете цельнокатаных колес вагонов и локомотивов. Определено, что максимальную погрешность в решение вносит малое количество узлов по толщине диска. Использование равномерной дискретизации в данном случае невозможно, так как приводит к очень большому числу степеней свободы модели. Относительно малое количество узлов по окружной координате не вносит существенной погрешности в решение задачи.